MIDTERM 2
MATH 430, SPRING 2014

There are 5 problems. All problems have equal value

Name:__________________________________

Problem	Score
1	
2	
3	
4	
5	
Total	

Problem 1. *Complete the following definitions:*

(1) *A set* $X \subset |\mathfrak{A}|$ *is definable iff:*

(2) *The Compactness Theorem states that:*

(3) *The Soundness theorem states that:*

(4) *There is a deduction from* Γ *to* ϕ *iff:*

(5) *A set of sentences* T *is a* **theory** *iff:*

Problem 2. *Let* $\mathfrak{A} = (\mathbb{N}; 0, +, \cdot)$. *Give a formula in the language of* $\mathfrak{A}$ *which defines the following. (Here the language includes* 0, 1, $+$, $\cdot$, $\forall$, $\exists$, *variables, equality and logical connectives).*

(a) $\{1\}$
(b) $\{3\}$
(c) $\{n \mid n \text{ is divisible by } 3\}$
(d) $\{\langle m, n\rangle \mid m \text{ and } n \text{ are coprimes}\}$

Problem 3. *Recall the two equivalent statements of the Completeness theorem:*

(a) If $\Gamma \models \phi$, then for some finite $\Delta \subset \Gamma$, $\Delta \models \phi$.

(b) If every finite subset of Γ is satisfiable, then so is Γ.

Prove that these two statements are equivalent. Then prove the Completeness theorem from the Compactness theorem.

Problem 4. *Show that if x does not occur free in any formula in Γ, then the set $S = \{\phi \mid \Gamma \vdash \forall x \phi\}$ is closed under modus ponens (i.e. whenever ϕ_1 and $\phi_1 \to \phi_2$ are both in S, then so is ϕ_2).*

Problem 5. *Suppose that σ has has arbitrarily large finite models. Show that σ has an infinite model.*